Intro to Proofs (Algebraic)

Proof: logical arguement that uses a sequence of statements to prove a conjecture

Theorem: A proven conjecture

Reasons can be: ① given information ② Definitions, ③ Previously proven theorems ④ mathematical propert

When writing an algebraic proof, you create a chain of logical steps that move from the hypothesis to the conclusion of the conjecture you are proving. By proving the conclusion is true, you have proven the original conjecture is true.

Hypothesis → • Definitions • Postulates • Properties • Theorems → Conclusion

When writing a proof, it is important to justify each logical step with a reason. You can use symbols and abbreviations, but they must be clear enough so that anyone who reads your proof will understand them.

Properties of Equality/Algebra Properties

Properties of Equality	Property
Addition Property of Equality	if $a=b$, then $a+c=b+c$
Subtraction Property of Equality	if $a=b$, then $a-c=b-c$
Multiplication Property of Equality	if $a=b$, then $a\cdot c=b\cdot c$
Division Property of Equality	if $a=b$, then $\frac{a}{c}=\frac{b}{c}$
Distributive Property	$a(b+c)=ab+ac$
Reflexive Property	$a=a$
Transitive Property	if $a=b$ and $b=c$, then $a=c$
Symmetric Property	if $a=b$, then $b=a$
Substitution Property	if $a=b$, then b can replace a
Associative Property	$a+(b+c)=(a+b)+c$
Commutative Property	$a+b=b+a$

I-Do: Name the property or equality that justifies each statement

1. If UV = KL and KL = 6, then UV = 6

Substitution

2. If m∠1 + m∠2 = m∠4 +m∠2, then m∠1 = m∠4

transitive

3. ∠ABC≅ ∠ABC

Reflexive

4. If $\frac{1}{2}m\angle D = 45$, then $m\angle D = 90$.

multiplication

I-Do: Use the property to complete each statement.

5. Reflexive Property of Congruence

$\angle TRS \cong$ ∠TRS

6. Substitution Property

If AB = 2, and AC = AB +BC, then AC = 2 + BC

We-Do: Identify the Property of equality that justifies the missing step(s) to solve the equation.

Example 1:

Equation	Steps
3x + (x − 8) = 12	Original Equation
4x − 8 = 12	Associative property of Addition
4x = 20	Addition property
x = 5	division

Example 2:

Equation	Steps
-6 = 3x − (x + 4)	Original Equation
-6 = 3x − x − 4	distributive
-6 = 2x − 4	Simplify or Combine Like Terms
-2 = 2x	addition
-1 = x	division
x = -1	symmetric

Bridging from Algebraic to Geometric

Practice #1:

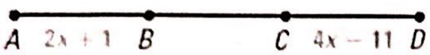
. GIVEN $PR = 46$

PROVE: $x = 7$

STATEMENTS	REASONS
$PR = 46$	Given
$PQ + QR = PR$	segment addition
$2x+5+6x-15=46$	substitution
$8x-10=46$	combine like term
$8x=56$	addition
$x=7$	division

Practice #2:

GIVEN $\overline{AB} \cong \overline{BC}, \overline{CD} \cong \overline{BC}$



PROVE: $x = 6$

STATEMENTS	REASONS
$\overline{AB} \cong \overline{BC}$	Given
$\overline{CD} \cong \overline{BC}$	Given
$\overline{AB} \cong \overline{CD}$	substitution
$\overline{AB} = \overline{CD}$	def. of congruent
$2x+1 = 4x-11$	substitution prop.
$2x+12=4x$	Addition prop.
$12 = 2x$	subtraction prop.
$6 = x$	division prop.
$x = 6$	symmetric prop.

What Is the length of $\overline{AB}$?

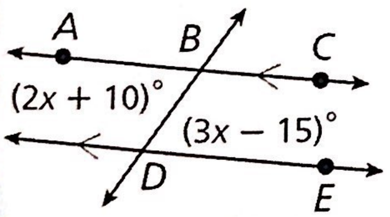
$\overline{AB} = 13$

What Is the length of $\overline{CD}$?

$\overline{CD} = 13$

Practice: #3

GIVEN: ∠ABD and ∠BDE are alternate interior angles.

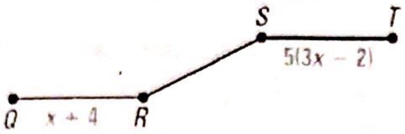


Prove: $m\angle DBC = 120°$

STATEMENTS	REASONS
∡ABD + ∡BDE are alt. int. angles	Given
∡ABD ≅ ∡BDE	def. of alt. int. ∡'s.
$2x+10 = 3x-15$	Substitution Prop
$2x+25 = 3x$	Addition Prop.
$25 = x$	Subtraction Prop.
$x = 25$	Symmetric Prop.

Practice: #4

GIVEN $\overline{ST} \cong \overline{SR}$, $\overline{QR} \cong \overline{SR}$



Prove: x = 1

STATEMENTS	REASONS
$\overline{ST} \cong \overline{SR}$	Given
$\overline{QR} \cong \overline{SR}$	Given
$\overline{ST} \cong \overline{QR}$	Transitive Prop
$5(3x-2) = x+4$	Substitution
$15x - 10 = x+4$	Distribution
$14x - 10 = 4$	Subtraction Prop
$14x = 14$	Addition Prop.
$x = 1$	Division Prop.

Practice: #5

GIVEN: $\overline{WX} = \overline{YZ}$
Y is the midpoint of $\overline{XZ}$.

Prove: $\overline{WX} = \overline{XY}$

STATEMENTS	REASONS
$\overline{WX} = \overline{YZ}$	Given
Y is the midpt. of $\overline{XZ}$	Given
$\overline{XY} \cong \overline{YZ}$	def. of midpoint
$\overline{XY} = \overline{YZ}$	def. of congrue
$\overline{WX} = \overline{XY}$	Substitution