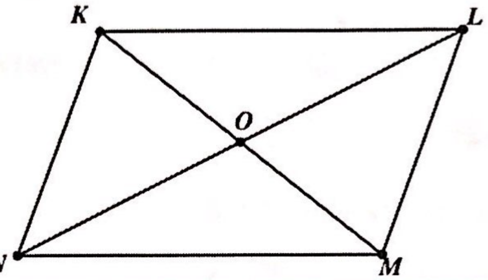
Geometric Proofs: Parallelograms

Properties of Parallelograms

A **parallelogram** is a type of quadrilateral that has **two pairs of opposite sides that are parallel.** Parallelograms are denoted by the symbol ▱. If a quadrilateral has two pairs of parallel, opposite sides, then it can be classified as a parallelogram.

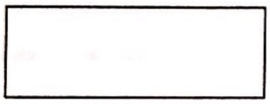
There are 5 theorems associated with PARALLELOGRAMS:

- Opposite sides are congruent
- Opposite angles are congruent
- Consecutive angles are supplementary
- Diagonals bisect each other
- Diagonals form two congruent triangles



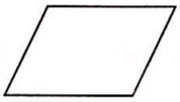
Parallelograms can be broken down into three more specific types of quadrilaterals with the same properties as parallelograms. The three specific types also have some of their own properties.

Rectangles



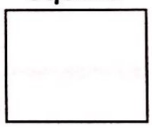
- **All properties of parallelograms**
- Diagonals are congruent
- Four right angles

Rhombus



- **All properties of parallelograms**
- Diagonals are perpendicular
- Diagonals bisect each other
- Four sides are congruent

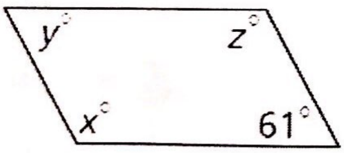
Square



- **All properties of parallelograms**
- Four right angles
- Four congruent sides
- Diagonals are congruent, perpendicular, and bisect each other

Check for Understanding: Solve the problems below.

1. Solve for x, y, and z.



Relationship: opposite ∡'s are ≅

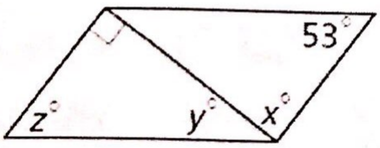
y = 61°

z = 119°

x + 61 = 180

x = 119°

2. Solve for x, y, and z.



Relationship: alt. int. ∡'s

z = 53°

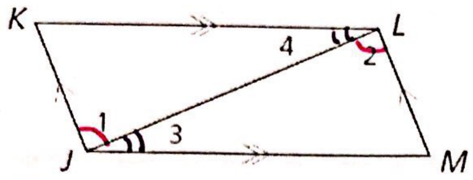
x = 90°

y = 37°

Proving Parallelograms

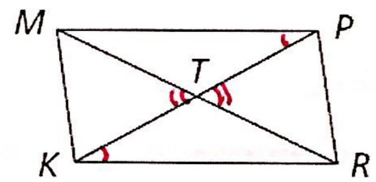
Yesterday, you explored 4 out of the 5 theorems associated with parallelograms. You learned that opposite sides are congruent, opposite angles are congruent, consecutive angles are supplementary, and diagonals bisect each other. It was mentioned that, in a parallelogram, diagonals form two congruent triangles, but you never really explored it. In the problem below, you are going to prove that a parallelogram forms two congruent triangles.

Given: JKLM is a parallelogram
Prove: $\Delta JKL \cong \Delta LMJ$



Statements	Reasons
1. JKLM is a parallelogram	1. Given
2. $\overline{KJ} \parallel \overline{LM}$, $\overline{KL} \parallel \overline{JM}$	2. def. of parallelogram
3. $\angle 4 \cong \angle 3$	3. Alternate Interior $\angle$'s are $\cong$
4. $\angle 1 \cong \angle 2$	4. alt. int. $\angle$'s are $\cong$
5. $\overline{JL} \cong \overline{JL}$	5. Reflexive prop.
6. $\Delta JKL \cong \Delta LMJ$	6. A-S-A congruence

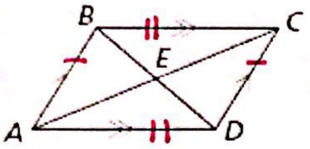
Using the picture at the right, answer the following questions about parallelogram MPRK. Justify your answer (using properties of parallelograms) for each question.



Statement	Reasons
a. $\angle MPR \cong$ $\angle RKM$	opposite angles are congruent
b. $\angle PRK \cong$ $\angle KMP$	CPCTC
c. $\overline{MT} \cong$ $\overline{RT}$	diagonals bisect each other
d. $\overline{PR} \cong$ $\overline{MK}$	parallel/opposite sides are congruent
e. $\overline{MP} \parallel$ $\overline{KR}$	opposite sides are parallel
f. $\overline{MK} \parallel$ $\overline{PR}$	opposite sides are parallel
g. $\angle MPK \cong$ $\angle RKP$	alt. int. $\angle$'s are $\cong$
h. $\angle MTK \cong$ $\angle RTP$	vertical angles are $\cong$
i. $m\angle MKR + m\angle PRK =$ 180	consecutive angles are supplementa

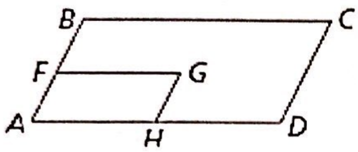
Proofs with Parallelograms

a. **Given:** ABCD is a parallelogram
Prove: $\angle BAD \cong \angle DCB$



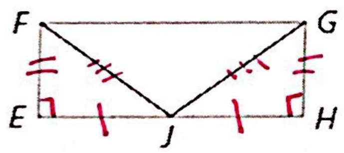
Statements	Reasons
1. ABCD is a parallelogram	1. Given
2. $\overline{AB} \cong \overline{CD}$	2. def. of parallelograms
3. $\overline{DA} \cong \overline{BC}$	3. def. of parallelograms
4. $\overline{AC} \cong \overline{AC}$	4. reflexive property
5. $\triangle ABC \cong \triangle CDA$	5. SSS
6. $\angle BAD \cong \angle DCB$	6. CPCTC

b. **Given**: ABCD and AFGH are parallelograms
Prove: $\angle C \cong \angle G$



Statements	Reasons
1. ABCD is a parallelogram	1. Given
2. AFGH is a ▱	2. Given
3. $\angle C \cong \angle A$	3. opp. $\angle$'s are $\cong$
4. $\angle A \cong \angle G$	4. opp. $\angle$'s are $\cong$
5. $\angle C \cong \angle G$	5. transitive property.

c. **Given:** EFGH is a rectangle, J is the midpoint of $\overline{EH}$.
Prove: $\triangle FJG$ is isosceles.



Statements	Reasons
1. EFGH is a rectangle.	1. Given
2. $\angle E$ & $\angle H$ are right angles.	2. def. of Rect.
3. $\angle E \cong \angle H$	3. ~~a~~ all right $\angle$'s are $\cong$
4. J is the midpoint of $\overline{EH}$.	4. Given
5. $\overline{EJ} \cong \overline{HJ}$	5. def. of midpoint
6. EFGH is also a parallelogram	6. def. of rect.
7. $\overline{FE} \cong \overline{GH}$	7. def. of parallelogram opp sides are $\cong$
8. $\triangle FJE \cong \triangle GJH$	8. S-A-S congruence
9. $\overline{FJ} \cong \overline{GJ}$	9. CPCTC
10. $\triangle FJG$ is isosceles	10. isosceles triangle theorem